Largest Laplacian eigenvalue predicts the emergence of costly punishment in the evolutionary ultimatum game on networks

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(Received 19 November 2008; revised manuscript received 29 September 2009; published 1 December 2009)

In recent years, there has been a growing interest in studying the role of costly punishment in promoting altruistic behaviors among selfish individuals. Rejections in ultimatum bargaining as a metaphor exemplify costly punishment, where the division of a sum of resources proposed by one side may be rejected by the other side, and both sides get nothing. Under a setting of the network of contacts among players, we find that the largest Laplacian eigenvalue of the network determines the critical division of players' proposals, below which pure punishers who never accept any offers will emerge as a phase transition in the system. The critical division of offers that predicts the emergence of costly punishment is termed as the selfishness tolerance of a network within evolutionary ultimatum game, and extensive numerical simulations on the data of the science collaboration network, and computer-generated small-world/scale-free networks support the analytical findings.

DOI: 10.1103/PhysRevE.80.066101 PACS number(s): 89.75.Hc, 02.50.Le, 87.23.Ge

In classical game theory, players are typically motivated by self-interest based on the assumption that all players are rational and uniquely attempt to maximize their own utility [1]. This self-interest hypothesis *de facto* rules out any altruistic behavior but which arises as a ubiquitous phenomenon in natural and social systems [2]. Are selfishness and altruism really mutually exclusive? Recent years have witnessed an enormous effort to explore the hidden mechanism that enables altruistic cooperation to be sustained among selfish individuals. Many behavioral experiments have been conducted among samples of diverse people, in which several metaphorical games with exceptionally simple rules are used to study human altruism, providing evidence that without insisting on self-interest, costly (or altruistic) punishment is, to a large extent, a key element in understanding the issue of widespread altruistic behaviors of human societies [3,11,15]. Costly punishment means that a punisher voluntarily pays a cost to incur a heavier loss (punishment) on the part of the selfish opponents (e.g., free-riders in the public goods, selfish proponents in the ultimatum game, or defectors in the prisoner's dilemma) [4]. Costly punishment has turned out to be an effective way of promoting altruism, which has apparently extended far beyond the self-interest hypothesis in economic rationality.

In contrast to classical game theory, which has difficulties explaining human altruism, evolutionary game theory provides a powerful framework to investigate the mechanisms of altruistic behaviors among selfish individuals [5]. In recent years, there has been an increasing interest in evolutionary game on networks, studying the effect of connectivity structures or interaction patterns represented by the network of contacts (NOCs) among individuals [6,7]. Many researchers have ascribed altruistic behaviors sustainable among structured population to the network effect, which is categorized into "network reciprocity," as one of the five fundamental rules to the evolution of cooperation [8].

We focus our attention here on the effect of network topology to the evolution of costly punishment within the ultimate game. As one celebrated example to highlight costly punishment [9], the ultimatum game assumes two players to split a sum of provided resources. One randomly chosen player (the proponent) proposes how to divide it, and his connected neighbor (the respondent) can either accept or reject the offer. If the respondent accepts, the resources are divided accordingly. If the respondent rejects, both players get nothing. Notice that a respondent's rejection is costly punishment, in which case the respondent who rejects an offer pays a cost, the imposition of punishment on the proponent resulting in a loss of his payoff that he could otherwise receive from the deal. In terms of classical game theory, a rational respondent motivated with his self-interest will accept any nonzero offer, even a minimal positive sum in the extreme case. Moreover, exempted from punishment, a rational proponent should therefore claim almost the whole resources for his own. However, according to a large number of extensive human studies, about half of the respondents reject unfair offers that are below 30% of the sum, and the average of accepted offers is around 50% [9]. That is, a human player in the ultimate game will reject unfair proposals, where costly punishment lifts sanctions on selfish players and restricts proponents from making excessively selfish offers. Then, a natural question arises, about how much a proponent offers to others that could incur costly punishment. We make an attempt to answer this question in the framework of evolutionary game on networks, and find that the critical minimal acceptable offer is inversely related to the largest Laplacian eigenvalue of the network on which the ultimatum game takes place.

In the ultimatum game, consider one randomly chosen player proposes an offer (as proponent) and his co-player (as respondent) selects a strategy from the binary choices: Accepting (A) or Rejecting (R) the offer. To tackle the difficul-

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ties shifting the roles of players, we first symmetrize the status of both players, assuming that the ultimatum game is a bidirectional play, i.e., the respondent in turn apply the same offer to the proponent. Without loss of generality, the sum of resource to split in every proposal is assumed to be 1, and each player claims $\bar{p}$ for himself and offers $1-\bar{p}$ for his co-player. If both players select R, they finalize no deals and get nothing. If one player selecting A encounters one R, then they reach one agreement in two deals from which the A receives $1-\bar{p}$ and the R receives $\bar{p}$ accordingly. If both select A, each accumulates 1 from the two agreed deals. Therefore, we have the following payoff matrix to describe the symmetry ultimatum game:

$$M = \begin{pmatrix} 1 & 1-\bar{p} \\ \bar{p} & 0 \end{pmatrix}. \tag{1}$$

Notice that the claim $\bar{p}$ describes the selfishness degree of players. The larger $\bar{p}$ is, the more selfish they are.

Consider that each player $\mathcal{P}_i$ $(i=1,2)$, 2) in the symmetry ultimatum game adopts a mixed strategy s_i on the two-dimension simplex

$$s_1 = \begin{pmatrix} r_1 \\ 1-r_1 \end{pmatrix} \quad \text{and} \quad s_2 = \begin{pmatrix} r_2 \\ 1-r_2 \end{pmatrix}, \tag{2}$$

where the mixed strategy s_i means that player $\mathcal{P}_i$ selects A with probability r_i, or R with probability $1-r_i$. Therefore, each player receives his payoff as

$$u_1 = s_1^T M s_2 \quad \text{and} \quad u_2 = s_2^T M s_1 \tag{3}$$

accordingly, where $(\cdot)^T$ denotes the transpose of a vector.

We take into account a population structure described by the network of contacts, where each player is located on a vertex of the network, and the interactions among players are represented by the edges of the NOCs. Each player, labeled from 1 to n, receives an accumulative payoff according to

$$u_i = \sum_{j\in N_i} s_i^T A s_j, \tag{4}$$

where $s_i=(r_i,1-r_i)^T$ is the mixed strategy of player $\mathcal{P}_i$, and N_i is the set of neighbors connected with $\mathcal{P}_i$ on the NOCs.

Motivated from the empirical results of the ultimate game among human players that about half of the respondents reject unfair offers from excessively selfish proponents [9], we address the issue of finding the criticality of selfishness degree, which indicates the emergence of pure costly punishers under the framework of evolutionary game theory. The role of empathy studied in [10] provides clues to understand the fairness induced by behavioristic ultimate games, where players (as proponents) make offers that are acceptable for themselves. Note that the principle of empathy can be applied *mutatis mutandis* to our model: along an evolutionary path toward empathy, players are hopefully evolved to the same accepting probability with respect to a given fixed offer [11]. Therefore, we design the accepting strategy updating dynamics to follow a consensus-protocol-type rule [12], and the consensus condition of achieving an identical accepting probability determines the critical selfishness degree p_c to separate two phases: (i) the population reaches a consensus of a positive accepting probability if the population selfishness degree $\bar{p}$ is below p_c; (ii) otherwise, such a consensus collapses, and a part of players in the system emerge as costly punishers who adopt the pure strategy of rejection.

Generally, a player cannot directly acquire the accepting probabilities of other players, which can be alternatively estimated from the normalized payoff of a player. Notice that the payoff function is positively correlated with the accepting probability, and has a maximum when all players set their accepting probabilities as one. Hence, we define the payoff ratio $\tilde{u}_i$ of player $\mathcal{P}_i$ as

$$\tilde{u}_i = \frac{\text{realized payoff of } \mathcal{P}_i}{\text{possible maximum payoff of } \mathcal{P}_i} = \frac{1}{k_i}\sum_{j\in N_i} s_i^T A s_j, \tag{5}$$

where k_i is player $\mathcal{P}_i$'s degree (the number of all connected neighbors) as the same as the total amount of resources that $\mathcal{P}_i$ has chance to share. Therefore, k_i is the possible maximal payoff of player $\mathcal{P}_i$ that he can receive from the NOCs, and $0\leq\tilde{u}_i\leq 1$ acts as an estimate of the accepting probability of player $\mathcal{P}_i$.

To fulfill the evolution of empathy following the framework of average-consensus problems, we propose that each player $\mathcal{P}_i$ adjusts his accepting probability by comparing his payoff ratio $\tilde{u}_i$ with the average level of all his connected neighbors. Intuitively, if $\mathcal{P}_i$ rejects offers with a larger (smaller) probability compared with his neighbors, he is hence inclined to increase (decrease) r_i to accept (reject) more. So we have the strategy updating dynamics

$$\dot{r}_i = \frac{1}{k_i}\sum_{j\in N_i} \tilde{u}_j - \tilde{u}_i, \tag{6}$$

where r_i is bounded within $[0, 1]$ to ensure that the mixed strategy remains a probability vector on the two-dimension simplex.

Substituting the payoff matrix (1) into Eq. (5) in the vector form, we have

$$\tilde{u} = r - \bar{p}\tilde{L}r, \tag{7}$$

where $\tilde{u}=(\tilde{u}_1,\tilde{u}_2,\cdots,\tilde{u}_n)^T$, $r=(r_1,r_2,\cdots,r_n)^T$, and $\tilde{L}$ is the (normalized) Laplacian matrix of the NOCs, whose diagonal entries are $\tilde{a}_{ii}=1$, and an off-diagonal entry $\tilde{a}_{ij}(i\neq j)$ equals to $-1/k_i$ if player $\mathcal{P}_j\in N_i$ is one neighbor of player $\mathcal{P}_i$, or otherwise 0. Moreover, it is very easy to verify that the row-sum of $\tilde{L}$ is 0. Therefore, Eq. (6) comes to

$$\dot{r} = -\tilde{L}\tilde{u} = -\tilde{L}(r-\bar{p}\tilde{L}r) = (\bar{p}\tilde{L}^2 - \tilde{L})r. \tag{8}$$

Note that all eigenvalues of $\tilde{L}$ are real and nonnegative denoted as $0=\tilde{\lambda}_1\leq\tilde{\lambda}_2\leq\cdots\leq\tilde{\lambda}_n\leq 2$, where the upper bound of eigenvalues is obtained by applying Gershgorin's disk theorem [13]. If $\bar{p}<1/\tilde{\lambda}_n$ holds, all eigenvalues of $\bar{p}\tilde{L}^2-\tilde{L}$ are nonnegative, which asserts the stability of Eq. (8). In this case, all players' accepting probability r_i eventually converge to the null space of $\bar{p}\tilde{L}^2-\tilde{L}$ spanned by the (right) zero eigenvector $[1,1,\ldots,1]^T$, so that $r_1=r_2=\cdots=r_n\triangleq r(\infty)$, i.e., the population reaches the consensus of an identical accepting probability $r(\infty)$ as

$$r(\infty) = \sum_{i=1}^{n} l_i \cdot r_i(0), \tag{9}$$

where $r_i(0)$ is the accepting probability of player $\mathcal{P}_i$ at the initial time, and $l=(l_1,l_2,\cdots,l_n)$ is $\tilde{L}$'s normalized left eigenvector corresponding to the zero eigenvalue, which is a nonnegative vector from Perron's theorem [13] So, in a population starting from a positive accepting probabilities $r_i(0)>0$, the strict consensus condition guarantees $r(\infty)>0$, indicating that in the consensus state, no pure costly punishers appear in the game system. On the other hand, if $\bar{p}>1/\tilde{\lambda}_n$, r diverge to reach the interval boundaries 0 or 1, respectively. Therefore, there exists the critical selfishness degree $p_c=1/\tilde{\lambda}_n$, and the condition of costly punishment emergence is

$$\bar{p} > p_c = 1/\tilde{\lambda}_n, \tag{10}$$

where $\tilde{\lambda}_n$ is the largest eigenvalue of $\tilde{L}$ [14]. The sufficiency and necessity of condition (10) have been hereby established.

Here, there arises another question that addresses the intuitive dependence of players' accepting probability r_i on their proponents' claim $\bar{p}$, which has not been reflected in our model so far. In particular, when the strict consensus condition holds, the final steady accepting probability $r(\infty)$ of the strategy updating dynamics should be expected to reflect such a pre-assumed dependence. Without loss of generality, we assume that the average of each respondent's initial accepting probability $<r_i(0)>=f(\bar{p})$ is a decreasing function of the fixed claim $\bar{p}$, where the angle bracket $\langle\bullet\rangle$ denotes the average over different realizations of $r_i(0)$. Notice that Eq. (9) implies that $r(\infty)$ is a convex combination of all players' initial accepting probability $r_i(0)$, from which we obtain $\langle r(\infty)\rangle=\sum_{i=1}^{n} l_i\cdot\langle r_i(0)\rangle=f(\bar{p})$. Therefore, the final consensus state $r(\infty)$ preserves the initial dependence of $r_i(0)$ on $\bar{p}$ as the strategy updating dynamics [Eq. (6)] evolves. It should be noted that $r_i(0)$'s initial distribution does not influence the critical selfishness degree. Furthermore, if $r(\infty)$ is required to be exactly $f(\bar{p})$, we can simply add a linear feedback term $-a[r_i-f(\bar{p})]$ to Eq. (6), where the tunable positive feedback gain a reflects the player's tendency of insisting on the pre-assumed accepting probability $f(\bar{p})$. Therefore, the critical population selfishness degree of the modified dynamics plus a linear feedback term is $p_c^{\text{feedback}}=(\tilde{\lambda}_n+a)/\tilde{\lambda}_n^2$. In the rest of this paper, we neglect the feedback term ($a=0$) for simplicity and without losing generality, since the case with a nonzero $a\neq 0$ can be similarly verified.

To visualize the effectiveness of evolutionary dynamics (6) and costly punishment emergence condition (10) in a structured population, we first make numerical simulations on a scale-free network generated by the Barabási-Albert (BA) model [16] with 10^4 vertices and 4×10^4 edges, the degree distribution of which reads a power law $P(k)\sim k^{-\gamma}$ with the fixed exponent $\gamma=3$. To initialize the simulation, assign each player $\mathcal{P}_i$ with an accepting probability r_i randomly distributed in $(0,1]$. At every round of the ultimatum game, each player $\mathcal{P}_i$ interacts with his connected neighbors in the NOCs receiving an accumulative payoff u_i according to Eq. (4), and then his payoff ratio $\tilde{u}_i$ and the derivative of his accepting probabilities r_i can be calculated according to Eqs. (5) and (6), respectively. To employ computer simulations, we discretize the continuous-time form of evolutionary dynamics (7) by choosing the time interval $\Delta=10^{-2}$ sec, and thus, player $\mathcal{P}_i$ adjusts his accepting probability r_i at the next time step as follows:

$$r_i(t+1) \to r_i(t) + \left(\frac{1}{k_i}\sum_{j\in N_i} \tilde{u}_j - \tilde{u}_i\right)\Delta, \tag{11}$$

where t is the time step. The accepting probability r_i is bounded in [0, 1] during the evolutionary process. Choose a Lyapunov candidate-function ρ as

$$\rho = \frac{1}{2}\tilde{u}^T\tilde{L}\tilde{u}. \tag{12}$$

The system evolves until a stationary state is reached, where the quantity ρ keeps invariant after sufficient time steps [17].

Varying the population's selfishness degree $\bar{p}$, which appears as a temperaturelike variable in the system, we observe the fraction of the pure punishers (with accepting probability $r=0$) and the stationary value of the Lyapunov candidate-function ρ. As shown in Fig. 1, there exists a phase transition point $p_c\approx 0.61$, which indicates the criticality of costly punishment emergence. When $\bar{p}<p_c$, $\rho=0$, and all players hold the same nonzero accepting probability [the left inset of Fig. 1(a)]. When $\bar{p}>p_c$, on the other hand, $\rho>0$, and the accepting probabilities of almost all the players diverge into two distinct groups: $r'=0$ and $r''=1$, respectively [the right inset of Fig. 1(a)], i.e., a fraction of players in the population emerge as pure (costly) punishers. As shown in the left inset of Fig. 1(b), the fraction of pure punishers in the population as a function of $\bar{p}$ exhibits a phase transition at the critical selfishness degree p_c. We calculate the largest Laplacian eigenvalue $\tilde{\lambda}_n^{BA}=1.64(5)$, and the predicted critical selfishness degree $1/\tilde{\lambda}_n^{BA}=0.60(8)$ according to Eq. (10), which agrees well with the numerical simulation. Besides, the simulation results obtained on the Watts-Strogatz (WS) small-world networks [18] are plotted in Figs. 1(c) and 1(d), which also shows a phase transition point near the calculated critical selfishness degree $1/\tilde{\lambda}_n^{WS}=0.62(8)$.

We then target the further numerical simulation at a sample of the science collaboration network (SCN) containing 9842 vertices and 37786 edges [19], where a scientist is a vertex in the network, and there is an edge between two scientists (represented by vertices) if they co-authored at least one paper [20]. The calculated selfishness degree $1/\tilde{\lambda}_n^{SCN}=0.58(3)$ is also verified by the emergence of pure punishers (or the quantity ρ) around $\bar{p}=0.6$ as shown in Fig. 2.

We further visualize the average accumulative payoff and the average payoff ratio of pure punishers (with $r_i=0$) and the remainders (with $r_i>0$) when the selfish degree $\bar{p}>p_c$. As shown in Fig. 3, the pure punishers who adopt the pure strategy of rejection have a relatively higher average payoff ratio compared to other players, and this gap enlarges as $\bar{p}$

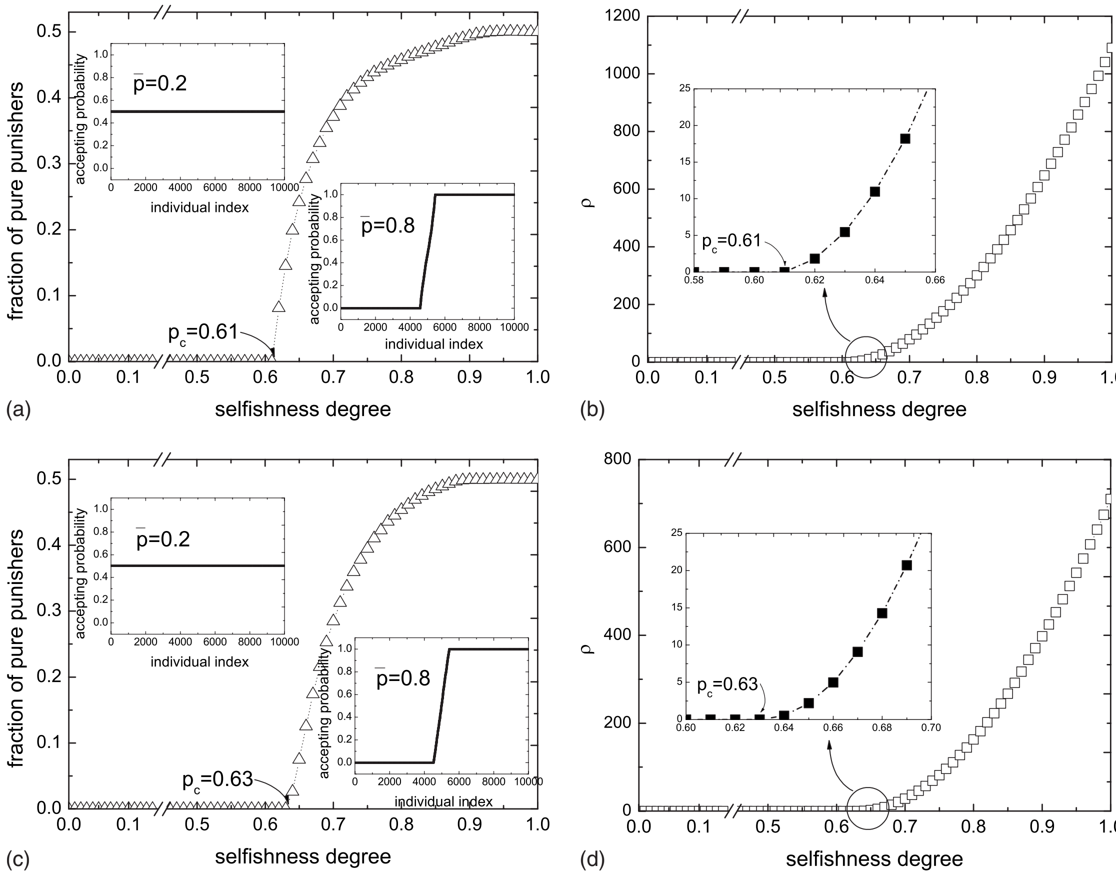

FIG. 1. The fraction of pure punishers who reject any offer versus the selfish degree $\bar{p}$, which exhibit a phase transition at the critical point (a)$p_c \approx 0.61$ for the BA scale-free network, and (c) $p_c \approx 0.63$ for the WS small-world networks (Insets: the distribution of accepting probabilities versus the players' individual index with r_i in the ascending order when $\bar{p}=0.2<p_c$ and $\bar{p}=0.8>p_c$, respectively). The curves of the stationary value of ρ versus the selfish degree $\bar{p}$ are plotted in (b) and (d) for both networks (Insets: the quantity ρ as a function of $\bar{p}$ near the critical selfish degree p_c). The population structures of (a)–(b) are the BA scale-free networks containing 10^4 vertices and 4×10^4 edges, and those of (c)–(d) are the WS small-world networks of the same size with the rewiring probability 0.25. Each point of the curves is averaged over 30 groups of different realizations.

increases, showing a positive correlation between costly punishment and the payoff ratio (other than accumulative payoff). Besides, when $p_c<\bar{p}<0.78$, pure punishers gain less accumulative payoff, implying that a winner does not punish [21]. However, when $\bar{p}>0.78$, pure punishers receive more than the remainders, indicating that costly punishment domi-

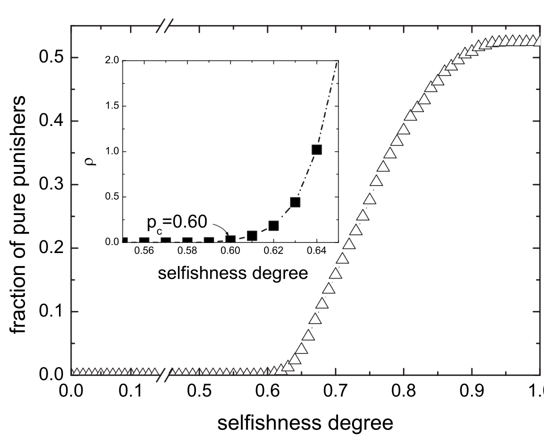

FIG. 2. The fraction of pure punishers and the quantity ρ (inset) versus the selfish degree $\bar{p}$ on the science collaboration network containing 9842 vertices and 37 786 edges (Ref. [19]). Each point of the curves is averaged over 30 groups of different realizations.

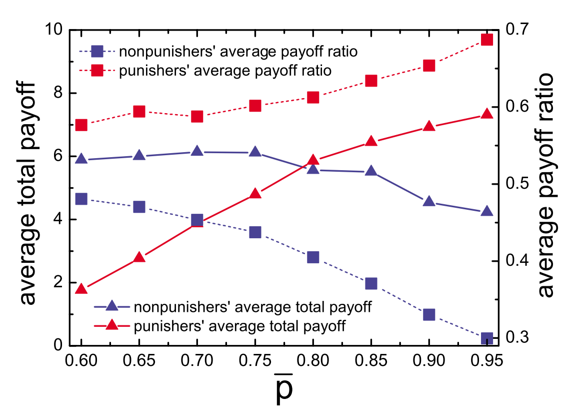

FIG. 3. (Color online) The curves of the average total payoff (solid lines with triangles) and the average payoff ratio (dashed lines with boxes) of pure punishers (red) and the remainders (blue) versus the selfish degree $\bar{p}$. Each point of the curves is averaged over 30 groups of different realizations.

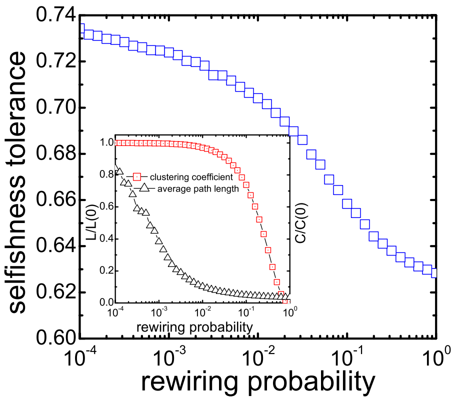

FIG. 4. (Color online) The curve of selfishness tolerance versus the rewiring probability. Inset: the normalized curves of the average path length and clustering coefficient versus the rewiring probability. The networks are generated by the WS model with size $n=2000$, and the average degree $\langle k\rangle=10$. Each point of the curves is averaged over 30 groups of different realizations.

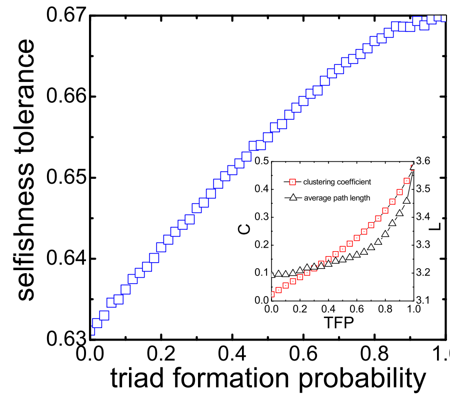

FIG. 5. (Color online) The curve of selfishness tolerance versus the triad formation probability (TFP). Inset: the curves of the average path length and clustering coefficient versus the triad formation probability. The networks are generated by the Holme-Kim model with size $n=2000$, the scale-free exponent $\gamma=3$, and the average degree $\langle k\rangle=10$. Each point of the curves is averaged over 30 groups of different realizations.

nates if the population has an extreme selfishness degree.

Denote $1/\tilde{\lambda}_n$ the selfishness tolerance of a network within the ultimatum game, which indicates the criticality of selfish degree to tolerate in a population connected by the network. We now further observe the topological influence of different categories of networks on which the ultimatum game is played.

We first visualize the selfishness tolerance of networks generated by the WS model [18], which transit from regular rings to small-world networks, and finally to completely random graphs [22] with increasing the rewiring probability from 0 to 1. As shown in Fig. 4, the selfishness tolerance of the WS networks in the ultimatum game is a decreasing function of the small-world rewiring probability. Moreover, as further observed in the inset of Fig. 4, the average path length and the clustering coefficient of the WS networks decrease with the increase in the rewiring probability, showing that selfishness tolerance is positively related with the small-world NOCs' average path length and clustering coefficient. Such a dependence also exists in the scale-free NOCs. With the clustering enhancement of the BA scale-free networks through the triad formations proposed by Holme and Kim [23], the scale-free networks keep the scale-invariant exponent $\gamma=3$ with tunable clustering coefficients. Figure 5 shows that the increase in selfishness tolerance depends on the increase in triad formations, where the clustering coefficient and average path length increase simultaneously (the inset of Fig. 5).

To summarize, in this paper, to study the emergence of costly punishment in the evolutionary ultimatum game on networks, we have proposed to fix each proponent's claim to reflect the selfishness degree of a homogeneous population, and to adjust each respondent's accepting probability in the ultimatum bargaining with a consensus-protocol-type updating rule to fulfill the evolution of empathy. We find that a fraction of players as pure costly punishers emerge after a phase transition with respect to the population selfishness degree, and the critical selfishness degree is analytically predicable with the largest Laplacian eigenvalue of the network. These findings indicate that the structural topology of a network suffices to determine the selfishness tolerance of a population in evolutionary ultimatum bargainings, and the roles of connectivity patterns of complex networks deserve more investigations in future.

The authors thank the anonymous reviewers for their constructive suggestions to help improve this work, and Dr. Zhihai Rong for helpful communications. This work was partly supported by the National Key Basic Research and Development Program (Grant No. 2010CB731403), the Natural Science Foundation of China (Grant No. 60874089), and the Shanghai Rising-Star Program (Grant No. 09QH1400200).

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[11] Note that the optimal strategy of a proponent with bounded rationality in the ultimate game is, obviously, proposing the minimal offer that his respondent could accept. In this case, the empathy of the proponent implies that he has the same minimal acceptable offer as his respondent, and hence, the whole population of players reach a consensus in their critical acceptable offers. We extend the concept of empathy in our model as a consensus in the accepting probabilities (with respect to a fixed offer $\bar{p}$) obtained by a population of players. As a result of evolution of empathy, the final configuration of the game system entails that all players have the same accepting probability, given $\bar{p}$ as a game parameter describing the selfishness degree in a homogeneous population.

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[14] Notice that the criticality $p_c = 1/\tilde{\lambda}_n \geq 0.5$ due to the largest Laplacian eigenvalue $\tilde{\lambda}_n \leq 2$, implying that hyperfair offers (offers above 50%) are always acceptable. However, recent behavioral experiments show that there exist hyperfair rejections in human societies (Ref. [15]). This counterintuitive result reflects some sociocultural factors such as a rejection of being subordinate, but which is out of the scope of this paper.

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[17] Notice the time derivative of Eq. (12) $\dot{\rho} = \tilde{u}^T \tilde{L} \dot{\tilde{u}} = \tilde{u}^T (\tilde{L}^2 - \bar{p} \tilde{L}^3) \tilde{u} < 0$ implies $\bar{p} < 1/\tilde{\lambda}_n$, which also yields the costly punishment emergence condition (10). Through a little computation, one can see that ρ describes the payoff ratios' differences among mutually contacting players in the population, where $\rho = 0$ implies all players reach a consensus of their payoff ratios. In the following numerical simulations of this paper, we terminate the evolutionary process when the variance of ρ in 500 time steps is less than 10^{-3}, which means that the game system has reached a stationary configuration.

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